Louis Joel Mordell's time in London

By

Ben Fairbairn

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Abstract. The celebrated number theorist Louis Joel Mordell spent around two and a half decades working in Manchester and for most of the rest of his career he was based in St John's College, Cambridge. There was, however, a brief period when he was based in London. The standard biographies of Mordell's life by and largely tend to overlook this period almost to the point of being deceptive about it. In this paper we will address this imbalance by discussing this chapter in Mordell's life in more detail.

1. Introduction

In 1969 just a couple of years before he died, Louise Joel Mordell gave a speech following a dinner in St John's College, Cambridge, in which he described the events of his life. Not long after he published a version of this speech in his 'Reminiscences of an Octogenarian Mathematician' [13] that we shall occasionally quote from and will henceforth refer to as Mordell's *Reminiscences*.

Following Mordell's death in 1972 J. W. S. Cassels wrote an extensive obituary of Mordell for the Royal Society [4] and to quote from it:

> As [Mordell's Remeiniscences] is readily available and gives a better impression of its author's personality than I could hope to do, I have resisted the temptation to quote in extenso in the hope that the reader will be tempted to look it out for himself.

Consequently these two sources between them provide a highly comprehensive account of Mordell's life and work. Again, we shall on occasion have a need to quote from this source and we shall henceforth refer to this as the *Cassels obituary*.

These two sources have comparatively little to say about the seven year period 1913-20 during which Mordell was working in London. Indeed, this brevity is, to some extent, a little mis-leading about what Mordell was doing at the time as we shall see and so this short note is an effort to try and rectify this imbalance.

This paper is organised as follows. In Section 2 we give a brief overview of Mordell's life leading up to when he first arrived in England in 1906 to provide some context. In Section 3 we give the main discussion of Modell's time in London before concluding in Section 4 with a brief biography of his later life for the sake of completeness.

2. Before London

To give some context to what follows we provide some background on Mordell's life from before he moved to London. We shall largely be following [4, 13, 14, 15].

He was born on January 28^{th} 1888 in Philadelphia, Pennsylvania, USA. His father, Phineas Mordell, was a Hebrew scholar and his mother was Annie Feller who were both Jewish Lithuanians that had emigrated to Philadelphia in 1881. Louis was the third of eight children (four sons and four daughters).

Having shown a great aptitude for mathematics at school, Mordell ambitiously tried for a place at the University of Cambridge in England, having been influenced by the ubiquity of Tripos examination question in the books he had (largely) taught

himself mathematics from. In December 1906 he travelled to England to attempt examinations for a scholarship and was successful.

3. London

3.1. **The standard references.** All that is said in Mordell's Reminiscences about this period is the following.

> In 1913, I was appointed a lecturer at Birkbeck College in London. There I stayed some seven years, except for some $2\frac{1}{2}$ years during the war in the statistical department of the Ministry of Munitions. I had during all this time continued my studies and researches, and I began to gain recognition. As my stature increased, my thoughts turned to professorships, and around 1919 and 1920, I applied unsuccessfully for two. In 1920, I decided that a change of scene would be welcome...

The Cassels obituary gives slightly more detail about this period of Mordell's life.

> When his time at Cambridge came to an end, Mordell wrote to his father that he had two offers of employment, on in London at £200 per annum and the other in Nova Scotia at £300. In a subsequent letter he told his father that he was taking the lower paid job because he felt that he would have more chance to develop as a mathematician in England than in Nova Scotia. The London job was, presumably, the lectureship at Birkbeck College which he occupied from 1913 until 1920 except for two years as a statistician at the Ministry of Munitions. One of his favorite anecdotes shows that for some of this period at least he still lived in Cambridge. He travelled by a certain train which should have got him to Birkbeck in time. But frequently the train arrived late. He pointed out the discrepancy between promise and performance to the Railway Company, who said that they would do something about it. And so they did: they adjusted the advertised time of arrival and, in consequence, the train now always arrived as advertised, but always too late for him.
>
> In May 1916 he married Mabel Elizabeth (1896–1971), the only daughter of Rosa and Joseph Cambridge, a small farmer who, so his grandfather recalls, pastured his stock on the greens of the eponymous town.

The Cassels obituary goes on to describe the more mathematical aspects of Mordell's work conducted during this period, a matter we will come back to later.

3.2. **The move to London.** The Cassells obituary's emphasis on the trains between Cambridge and London gives the impression that he spent this period living in Cambridge and commuting. It is, however likely that the move to London took place much earlier than this. The 1911 census for England and Wales lists him as boarding not in Cambridge, but at 16 Leinster Square London W [6] — see Figure 1.

He did not, however, spend his entire time in London at this address. On the contrary, much of the correspondence Mordell exchanged with G. H. Hardy during this period is all clearly addressed "38 Drakefield Rd, Balham, S. W. 17" a typical example being given in Figure 2.

Very few letters in this early correspondence have precise dates on them making it difficult to tell when he moved to this address. From the contents of what survives of

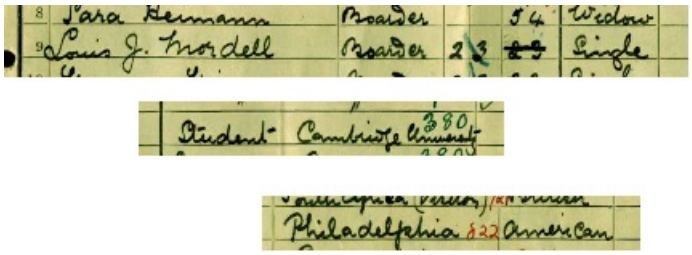


FIGURE 1. Mordell's entry in the 1911 census of England and Wales.

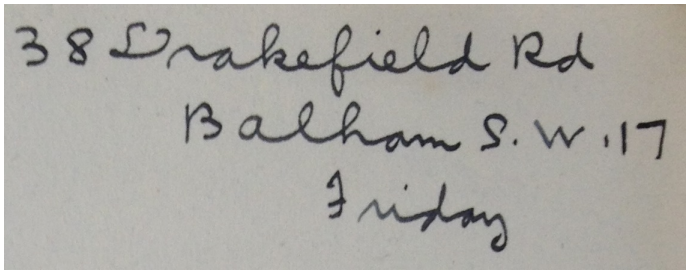


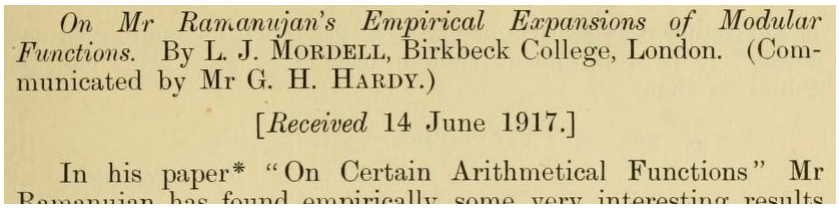
FIGURE 2. Mordell's London address.

the correspondence from this time, it is likely that most of it was exchanged during the period 1919–20. This is corroborated by the fact that more than one of the letters Mordell sent ask Hardy for job references for applications to Aberystwyth and Holloway College (now Royal Holloway, University of London — more on this later).

Interestingly, closer inspection of his correspondence from this period makes it clear that at least some of it must be somewhat older than previously thought. One piece in particular, dated simply "Feb 22nd", begins as follows.

> Dear Mr Hardy
>
> I have read with much interest Mr Ramanujan's paper "On Certain Arithmetical Functions". The results of Article 18 seem very difficult to prove. On the contrary, the results of Articles 20, 21, 22, 23, 28 (except results 159 & 162 where C_p is not defined) do not seem to present much difficulty.

It goes on to discuss mathematical technicalities, most notably Ramanujan's conjecture concerning the quadratic form $x^2 + 2y^2 + 10z^2$. It is clear that he has read this paper in some detail and spent much time thinking about its contents. It is impossible to know if Mordell is referring to a copy he has seen that has already been published or if he is working from an earlier preprint version (it is unclear how much direct contact Mordell may have had with Ramanujan or if Hardy consulted Mordell concerning Ramanujan's work but it is not inconceivable that this happened. Indeed the general tone of this letter suggests that it may even be a referee report of the paper.) The paper in question [16] appeared in 1916 so it is unlikely that this letter was written much earlier than 1915. One thing is, however, certain: he had definitely read [16] by the summer of 1917, the year in which he published [11] (it is listed as 'Received 14 June 1917' — see Figure 3.) The paper explicitly begins

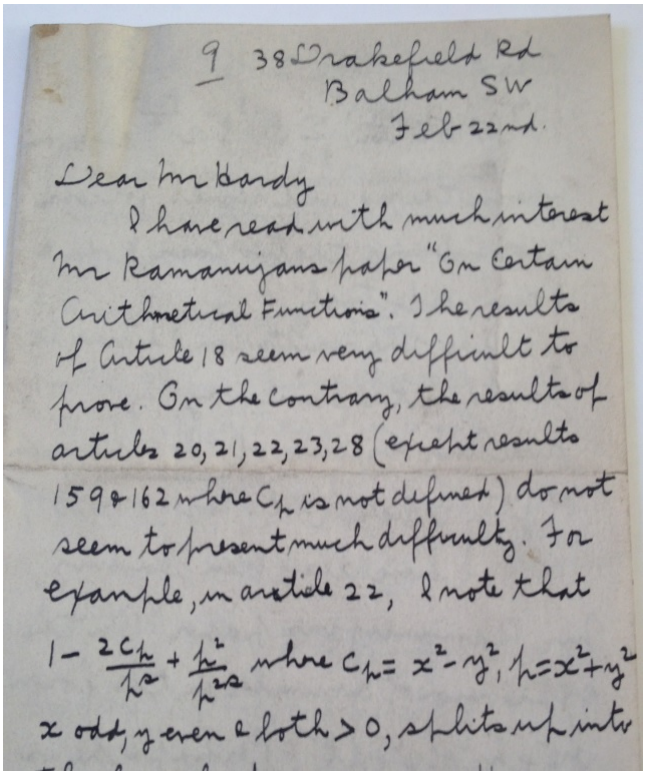


On Mr Ramanujan's Empirical Expansions of Modular Functions. By L. J. Mordell, Birkbeck College, London. (Communicated by Mr G. H. Hardy.)

[*Received* 14 June 1917.]

In his paper* "On Certain Arithmetical Functions" Mr Ramanujan has found empirically some very interesting results

FIGURE 3. The start of Mordell's 1917 paper on Ramanujan's tau function.



9 38 Drakefield Rd
Balham SW
Feb 22nd.

Dear Mr Hardy

I have read with much interest Mr Ramanujans paper "On Certain Arithmetical Functions". The results of Article 18 seem very diffinilt to prove. On the contrary, the results of articles 20, 21, 22, 23, 28 (except results 159 & 162 where C_n is not defined) do not seem to present much difficulty. For example, in article 22, I note that

$1 - \frac{2C_n}{n^2} + \frac{n^2}{n^{2s}}$ where $C_n = x^2 - y^2$, $n = x^2 + y^2$

x odd, y even & both > 0, splits up into

FIGURE 4. The beginning of the letter Mordell wrote to Hardy concerning Ramanujan's "On Certain Arithmetical Functions".

> In his paper "On Certain Arithmetical Functions" Mr Ramanujan has found empirically some very interesting results as to the expansions of functions which are practically modular functions ...

suggesting the above correspondence almost certainly predates the publication of [16] — see Figure 4. In particular, since this letter is addressed as being from his London residence his move to Drakefield Road must have happened before this time.

3.3. **Birkbeck College.** As we have already discussed, Mordell's position in London was a lectureship at Birkbeck College (now Birkbeck, University of London). Whilst the college is today located in Bloomsbury it did not move to its current premises until the mid 1950s. During the period in which Mordell was working there, the main buildings of the college were based further to the southeast of London in the Breams building in Holborn just south of Chancery Lane where it still stands today.

It is unlikely that Birkbeck had many teaching premises besides this owing to the college's financial position at the time. In its early days, the college was not just an educational establishment but provided other amenities such as financial services. In 1911, following a run on its deposits, the Birkbeck Bank collapsed and spectacularly so, ultimately being taken over by the London County & Westminster Bank (today, a constituent of NatWest) [7]. The hardship associated with the First World War can hardly have helped.

At the time, as now, Birkbeck College taught classes in the evenings in an effort to give an education to ordinary working Londoners.

It is clear from Birkbeck's official history [3] that this was a period of great change for the college. Around the time Mordell started at the college, a review of the University of London by Lord Haldane recommended that the college become part of the University of London. This process would not be completed until, coincidentally, around the time Mordell left the college in 1920, the process having been delayed by the war.

During the First World War lectures were introduced in the College on military subjects to meet the new demand. One in four of the staff and students who enlisted were ultimately killed during the conflict. Birkbeck also offered free education to Belgian refugees.

It is not clear how well Mordell himself got along with his mathematical colleagues at Birkbeck, though some of his correspondence hints that he may not have got along very well. In his later correspondence from the period when asking for references he would often indicate that Hardy alone was familiar with his work. To quote from one letter:

> I am applying for the Prof. of Pure Maths. at Aberystwyth (applications no later than July 24), and should like some one to say a few words about my mathematical work. You are the only mathematician I know personally, who has taken any interest in my work. Would it be too great a liberty for me to ask for a statement or a few words about my work?

Another, when asking for a reference to apply for a job at Holloway College, goes on:

> I am at a loose end when it comes to finding people who are intimately acquainted with my work.

We shall return to this point later.

3.4. **War work.** As we heard in section 3.1 Mordell spent around 2-2$\frac{1}{2}$ years in the 'statistical department of the Ministry of Munitions'. This work commenced in 1916 but it is difficult to find much record of what Mordell's work actually involved in practice, however there is one intriguing clue among his correspondence sent later in his time in London. One can only assume writing paper was expensive and in short supply at the time. Consequently, much of the correspondence that Mordell both sent and received is written on the back of scraps of paper previously used for other purposes. Examples include already marked undergraduate exams on Euclidean geometry; one letter from the 1940s is written on a flyer advertising a joint meeting of the London Mathematical Society with the Institute of Physics and another is on the back of an English undergraduate essay ("The United States of Europe" by G. L. Watson). Indeed there are several instances of letter-headed paper from previous appointments being used with the letter-heading crossed out and a new address written in its place (several of Hardy's letters have a neatly printed "New College Oxford" letter-head crossed out with "Trinity College Cambridge" scribbled underneath.)

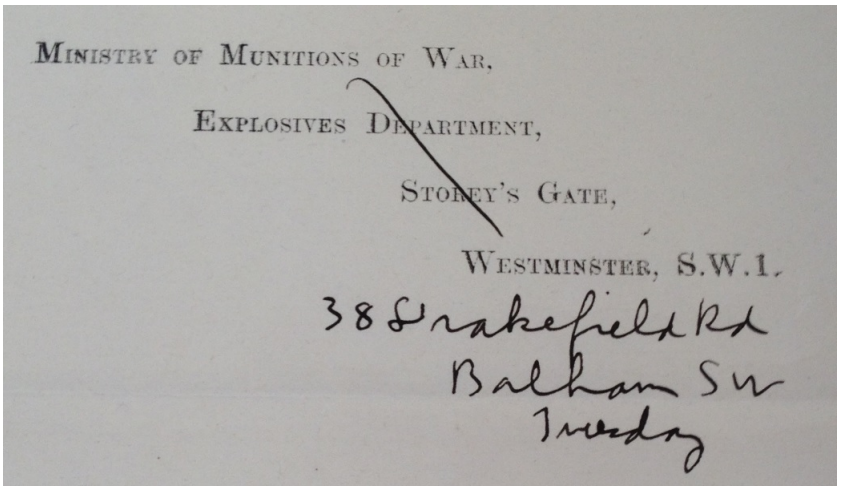


FIGURE 5. Letter-headed paper related to Mordell's war work.

In particular there is one instance of a letter Mordell wrote to Hardy on paper dating from his war work with the address at the top of the page (dutifully crossed out) being "Ministry of Munitions of War, Explosives Department, Stoney's Gate, Westminster, S. W. 1" suggesting that it was this specific division that Mordell worked in — see Figure 5.

In contrast to the Second World War there is little in the literature on the work of mathematicians during the First World War. What little has appeared, most notably [1, 2], has tended to emphasise computational efforts, mostly in ballistics, with little on the employment of mathematicians as statisticians, so it is difficult to say much more on this subject.

3.5. **Mordell's Mathematics.** During the period 1913–20, Mordell successfully published twenty one papers on a variety of different topics. His first publication during this period [10] was the publishing of his Smith's prize essay written during his time as a student at Cambridge on the topic of his now eponymous equation.

We have already mentioned the most significant piece of work published during this period [11]. What was the mathematical content of this paper? In [16] Ramanujan had defined the function τ as follows. We write $r := \exp(2\pi iz)$ where z lies on the upper half plane $\{w \in \mathbb{C} \,|\, \Im(w) > 0\}$ and define τ with the equation

$$r[(1-r)(1-r^2)(1-r^3)\cdots]^{24} = \sum_{n=1}^{\infty} \tau(n) r^n.$$

Ramanujan had conjectured in [16] that this function three specific properties:

- $\tau(mn) = \tau(m)\tau(n)$ whenever $\gcd(m,n) = 1$ (that is, τ is a multiplicative function);
- $\tau(p^{n+1}) = \tau(p)\tau(p^n) - p^{11}\tau(p^{n-1})$ for p prime such that $n > 0$ and
- $|\tau(p)| \leq 2p^{11/2}$ for all primes p.

In [11] Mordell successfully proved the first two of these conjectures. In retrospect, this work has been widely acknowledged to contain much of the modern theory Hecke operators, rediscovered by Hecke some twenty years later in [8, 9]. As an indication of how difficult it was to prove these results, the third conjecture wasn't verified until as recently as 1974 in Pierre Deligne's work on the Weil conjectures [5] that he went on to win the Fields medal for.

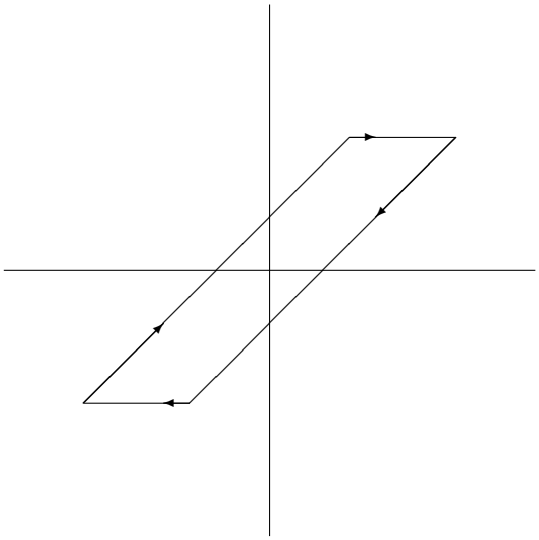


FIGURE 6. Mordell's contour.

One of his more quirky results the Mordell proved appeared towards the end of this period. There are several methods of calculating $\int_{-\infty}^{\infty} e^{-t^2} dt$ but for a long time nobody was sure how to calculate it using complex variable theory. One approach to this was discovered by Mordell in [12] using the extremely unusual contour depicted in Figure 6.

4. After London

After leaving Birkbeck College in 1920 Mordell commenced a lecturing position at Manchester College of Technology (later UMIST and ultimately absorbed into the University of Manchester). He left in 1922 to take up a readership at the University of Manchester. The next 23 years turned out to be the most successful of his career: he became Fielden Chair of Pure Mathematics in 1923; elected a Fellow of the Royal Society in 1924; awarded the De Morgan Medal from the London Mathematical Society in 1941 and became their president in 1943.

One aspect of Mordell's time in Manchester helps put his time in London into perspective. We remarked towards the end of Section 3.3 that Mordell may have been somewhat unhappy in London. This could simply be a reflection of an aspect of his character that disliked staying in on place. Some correspondence with Hardy during this later period discusses applying for the Sadleirian Chair in Cambridge as early as 1931. This is corrobarated by the fact that in later life he travelled extensively visiting colleagues across the globe at almost every opportunity.

In 1945 he moved back to St John's College, Cambridge, where he finally took up the Sadleirian Chair. He retired in 1953, though continued to be very mathematically active, and remained in Cambridge until his death on Sunday March 12^{th} 1972.

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Ben Fairbairn, Department of Economics, Mathematics and Statistics, Birkbeck, University of London, Malet Street, London WC1E 7HX, United Kingdom
E-mail address: b.fairbairn@bbk.ac.uk